INFORMATION SOCIETY TECHNOLOGIES
(IST)
PROGRAMME

Project IST-2001-33562 MoWGLI

Prototype n. D2.e - D2.f
Presentational Stylesheets

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Project Acronym: MoWGLI
Project full title: Mathematics On the Web: Get it by Logic and Interfaces
Proposal/Contract no.: IST-2001-33562 MoWGLI

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1 Overview

This document is concerned with the presentational stylesheets that are used for passing from the intermediate representation of mathematical proofs, as explained in document D2c-D2d, to the final presentation.

2 Introduction

In the Preliminary Report on Application Scenarios and Requirement Analysis, we already pointed out the use of XSLT stylesheet as the most natural approach for the re-mathematization of the logical, symbolic content of the information. The broad goal was that of developing coherent and well maintained libraries of notational stylesheets, publicly available on the web and freely reusable by any interested party.

Form the architectural point of view, we identified four different transformation phases of a document from the internal representation in some Proof Assistant Application to its final rendering: exportation, transformation, presentation, and rendering. Fig. 1, taken form Report D1a, gives a sketch of the overall architecture of these phases.

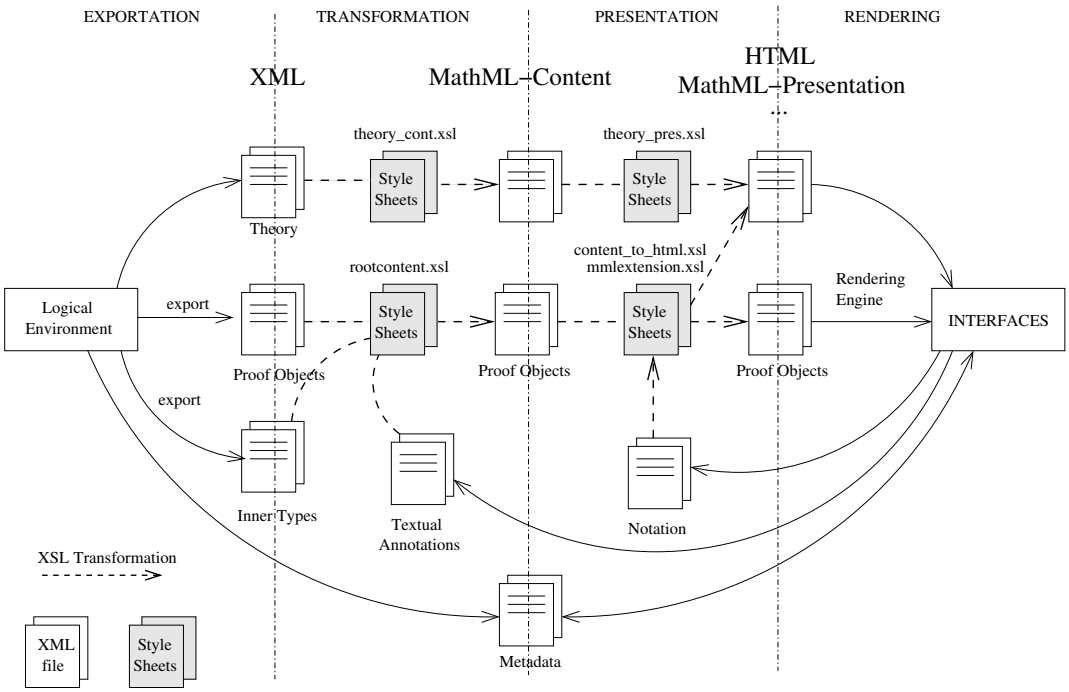

Figure 1: Transformation Phases and main stylesheets.

The first two phases (exportation and transformation) have been already and successfully completed (see deliverables D2.a and D2.c-D2.d). This report is about the third phase: the presentation of proofs and formulae. In this phase the OMDoc/MathML-Content files are transformed to HTML and MathML-Presentation.

3 A transformation example

Extending the example given in D2.c-D2.d, we consider the OMDoc representation of a proof of $0 \leq n$ for all natural numbers n. It is presented in appendix A. For readability most attributes are ommited.

The root element is an `omdoc:proof` (lines 2-161). The proof is made of just one big `omdoc:derive` step (lines 5-160) that concludes that $\forall n : nat.0 \leq n$ (lines 6-25) by discharging (line 26) the declaration of the arbitrary but fixed n of type *nat* (lines 28-35). More precisely, the `omdoc:method Intros+LetTac` is used to generalize the thesis of its only sub-proof (lines 27-158) by discharging all the variables and hypotheses in its context (i.e. all its `omdoc:declaration` and `omdoc:hypothesis` children, in this case only "$n : nat$".). In our example, the sub-proof starts with the declaration (lines 28-35) and is concluded by a final `omdoc:derive` step (lines 36-157) that proves that $0 \leq n$ (lines 37-47) by induction over n (line 48). The proof by induction has two subproofs (lines 70-93 and 94-155) which correspond to the base case and the inductive step. Remark that both have an `omdoc:label` child (lines 71 and 95).

B shows the same proof in MathML-Presentation. Again most attributes are ommited. Observe that:

- Information stored in OMDoc elements, is now shown in texts. See for example line 76: `<m:mtext mathcolor="red">we proceed by induction on </m:mtext>`, corresponding to line 48 in A.
- To mathematical objects its rendering is assigned. This is done either by Unicode, or by already existing appropriate MathML-Presentation tags. The former is shown in line 23:
  `<m:mo mathcolor="Blue">&#x2200</m:mo>`, producing "∀" and corresponding to line 9 in A.
- Several subproofs can be folded and unfolded.See lines 114-168 (70-93 in A).

Fig. 2 shows the HTML rendering of the same proof. It is possible to follow hyperlinks to the lemmas and definitions that are involved. The fully unfolded version can be seen in Fig. 3.

4 The transformation of mathematical items

The transformation of the proofs is mainly done by looking at the `omdoc:proof` tags and considering their `xref` attributes. These can be `Exact`, `Apply` or `Case`, to name a few, resulting for example in the texts "consider", "by" and "by cases on" respectively. But also other tags like `omdoc:hypothesis`, `omdoc:derive` and `omdoc:declaration` are taken into account.

The transformation of mathematical objects, like constants and variables, starts from their MathML-Content representation. Remember that in the second phase elementary MathML-Content tags or new `m:csymbol` elements replace the explicit names of the objects. The constant with the uri cic:Algebra/algebra/CSetoids/cs_ap.con, for instance, is in most cases replaced by `<m:csymbol>ap</m:csymbol>`. In the third phase, the presentational phase, this changes in "#". Also in the latter phase the order of the terms is fixed, so that we will see (0#1) instead of (#01). In practice, however, these phases are quite entangled: stylesheets for the second and the third phase can be automatically generated from just one file.

```
TYPE=
  ∀n:nat.O≤n
BODY=
  assume n:nat
    we proceed by induction on n to prove O≤n
      case O ⇒
        the thesis becomes O≤O
          Proof of O≤O
      case S : n0:nat ⇒
        the thesis becomes O≤ 1+n0
        (IHn) by induction hypothesis we know O≤n0
          Proof of O≤ 1+n0
    we proved O≤n
  we proved ∀n:nat.O≤n
```

Figure 2: HTML rendering of the folded proof

```
TYPE=
  ∀n:nat.O≤n
BODY=
  assume n:nat
    we proceed by induction on n to prove O≤n
      case O ⇒
        the thesis becomes O≤O
          by (le_n .)
          we proved O≤O
      case S : n0:nat ⇒
        the thesis becomes O≤ 1+n0
        (IHn) by induction hypothesis we know O≤n0
          by (le_S . . IHn)
          we proved O≤ 1+n0
    we proved O≤n
  we proved ∀n:nat.O≤n
```

Figure 3: HTML rendering of the unfolded proof

The transformation splits into two pipelines: one going to HTML, the other to MathML-Presentation.

4.1 The transformation into HTML

Figure 4 discribes the organization of the XSLT stylesheets in charge of transforming mathematical items in the intermediate representation into its presentation in HTML.

4.1.1 content_to_html

The file `content_to_html` includes the following ten stylesheets with between brackets the libraries with which they are concerned:

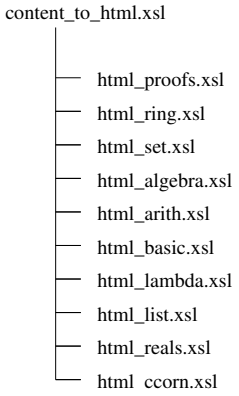

Figure 4: Inclusion structure for XSLT stylesheets to HTML

- `html_proofs.xsl` Proofs.
- `html_ring.xsl` Polynomials (Coq standard library, Rings).
- `html_set.xsl` $\in$, $\notin$, $\cap$, $\cup$, ... (Coq standard library, Sets).
- `html_algebra.xsl` $\leq$, $\#$, $\neq$, $+$, ... (CCoRN library).
- `html_arith.xsl` $<$, $\leq$, $>$, $+$, $-$, ... (Coq standard library, Arith).
- `html_basic.xsl` $\vee$, $\wedge$, $\forall$, ... (Coq standard library, Basic).
- `html_lambda.xsl` $\subseteq$, $\leftarrow$, $\rightarrow$, $\Rightarrow$, ...
- `html_list.xsl` $[]$, $\in$, $\subseteq$, ... (Coq standard library, Lists).
- `html_reals.xsl` $\leq$, $<$, $-$, $\geq$, ... (Coq standard library, Reals).
- `html_ccorn.xsl` ε, η, $1/2$, $(1/3)^n$ ... (CCoRN library, some variables).

Except `html_proofs.xsl`, `html_rings.xsl` and `html_ccorn.xsl`, these files are created from the same file as the relating stylesheets for the second phase. As said before, the second phase starts from files with explicit names of the mathematical objects. This explains why even the structure of the stylesheets of the presentational phase still reflects the origin of the mathematical objects.

4.2 The transformation into MathML-Presentation

Figure 5 discribes the organization of the XSLT stylesheets in charge of transforming mathematical items in the intermediate representation into its presentation in MathML-Presentation. Notice that the structure differs completely from the inclusion structure to HTML (Figure 4). Because at present the used stylesheets are hand-made and not, like the ones to HTML, generated from the same file as the stylesheets for the second phase, the structure does not reflect the origin of the mathematical objects.

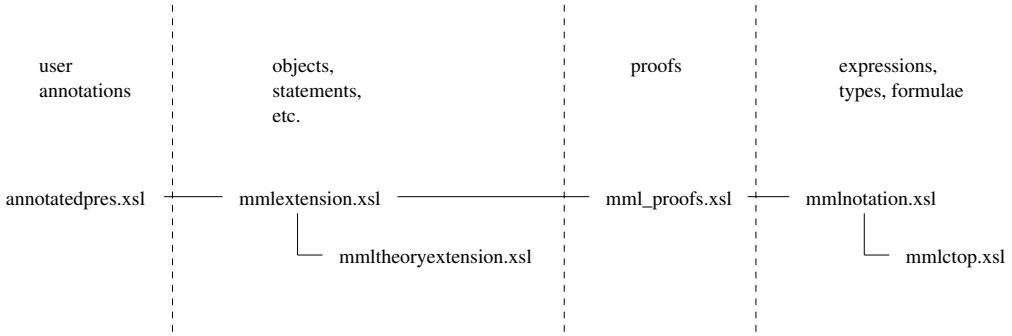

Figure 5: Inclusion structure for XSLT stylesheets to MathML-Presentation

4.2.1 annotatedpres.xsl

Proofs of the library may have user-supplied presentational annotations. If this is the case they are directly rendered. The file `annotatedpres.xsl` imports `mmlextension.xsl`, which is essentially the root of the presentation phase.

4.2.2 mmlextension.xsl

The file `mmlextension.xsl` is the root of the presentation phase. It takes care of the presentation of lambda-calculus and imports three stylesheets:

- `mmlnotation.xsl`
- `mmltheoryextension.xsl`
- `mml_proofs.xsl`

The file `mmltheoryextension.xsl` is a minor extension of `mmlextension.xsl`. The file `mml_proofs.xsl` deals with proofs.

4.2.3 mmlnotation.xsl

The file `mmlnotation.xsl` supplies the presentation of some basic mathematical notions. It imports `mmlctop.xsl`. The latter file does most of the transformation from MathML-Content to MathML-Presentation.

A OMDoc representation

```
<?xml version="1.0"?>
<omdoc:proof
  xmlns:omdoc="http://www.mathweb.org/omdoc"
  xmlns:m="http://www.w3.org/1998/Math/MathML">
  <omdoc:derive>
    <omdoc:FMP>
      <m:math>
        <m:apply>
          <m:csymbol>forall</m:csymbol>
          <m:bvar>
            <m:ci>n</m:ci>
              <m:type>
                <m:ci>nat</m:ci>
              </m:type>
          </m:bvar>
          <m:apply
          xmlns:i="http://www.cs.unibo.it/helm/InputNotationalElements"
          xmlns:o="http://www.cs.unibo.it/helm/OutputNotationalElements">
               <m:leq />
            <m:ci>O</m:ci>
            <m:ci>n</m:ci>
          </m:apply>
        </m:apply>
      </m:math>
    </omdoc:FMP>
    <omdoc:method xref="Intros+LetTac">
      <omdoc:proof id="i0">
        <omdoc:declaration>
          <omdoc:presentation><omdoc:use format="pmml">n</omdoc:use></omdoc:presentation>
          <omdoc:type>
            <m:math>
              <m:ci>nat</m:ci>
            </m:math>
          </omdoc:type>
        </omdoc:declaration>
        <omdoc:derive>
          <omdoc:FMP>
            <m:math>
              <m:apply
              xmlns:i="http://www.cs.unibo.it/helm/InputNotationalElements"
              xmlns:o="http://www.cs.unibo.it/helm/OutputNotationalElements">
                <m:leq/>
                <m:ci>O</m:ci>
                <m:ci>n</m:ci>
              </m:apply>
            </m:math>
          </omdoc:FMP>
          <omdoc:method xref="By_induction">
            <m:math
            xmlns:helm="http://www.cs.unibo.it/helm"
            xmlns:omdoc="http://www.mathweb.org/omdoc">
              <m:ci>n</m:ci>
            </m:math>
            <m:lambda
            xmlns:helm="http://www.cs.unibo.it/helm">
              <m:bvar>
```

```
57                  <m:ci><m:msub><m:mi>n</m:mi><m:mn>0</m:mn></m:msub></m:ci>
58                  <m:type>
59                    <m:ci>nat</m:ci>
60                  </m:type>
61                </m:bvar>
62                <m:apply
63                xmlns:i="http://www.cs.unibo.it/helm/InputNotationalElements"
64                xmlns:o="http://www.cs.unibo.it/helm/OutputNotationalElements">
65                  <m:leq/>
66                  <m:ci>0</m:ci>
67                  <m:ci><m:msub><m:mi>n</m:mi><m:mn>0</m:mn></m:msub></m:ci>
68                </m:apply>
69              </m:lambda>
70              <omdoc:proof id="i88" style="arity: 0">
71                <omdoc:label>0</omdoc:label>
72                <omdoc:derive>
73                  <omdoc:FMP>
74                    <m:math>
75                      <m:apply
76                      xmlns:i="http://www.cs.unibo.it/helm/InputNotationalElements"
77                      xmlns:o="http://www.cs.unibo.it/helm/OutputNotationalElements">
78                        <m:leq/>
79                        <m:ci>0</m:ci>
80                        <m:ci>0</m:ci>
81                      </m:apply>
82                    </m:math>
83                  </omdoc:FMP>
84                  <omdoc:method xref="Apply">
85                    <m:math>
86                      <m:ci>le_n</m:ci>
87                    </m:math>
88                    <m:math>
89                      <m:ci>.</m:ci>
90                    </m:math>
91                  </omdoc:method>
92                </omdoc:derive>
93              </omdoc:proof>
94              <omdoc:proof id="i16" style="arity: 2">
95                <omdoc:label>S</omdoc:label>
96                <omdoc:declaration>
97                  <omdoc:presentation>
98                    <omdoc:use format="pmml">
99                      <m:msub><m:mi>n</m:mi><m:mn>0</m:mn></m:msub>
100                   </omdoc:use>
101                 </omdoc:presentation>
102                 <omdoc:type>
103                   <m:math>
104                     <m:ci>nat</m:ci>
105                   </m:math>
106                 </omdoc:type>
107               </omdoc:declaration>
108               <omdoc:hypothesis inductive="yes">
109                 <omdoc:label>H</omdoc:label>
110                 <omdoc:FMP>
111                   <m:math>
112                     <m:apply
113                     xmlns:i="http://www.cs.unibo.it/helm/InputNotationalElements"
114                     xmlns:o="http://www.cs.unibo.it/helm/OutputNotationalElements">
```

```
                      <m:leq/>
                      <m:ci>O</m:ci>
                      <m:ci><m:msub><m:mi>n</m:mi><m:mn>0</m:mn></m:msub></m:ci>
                    </m:apply>
                  </m:math>
                </omdoc:FMP>
              </omdoc:hypothesis>
              <omdoc:derive>
                <omdoc:FMP>
                  <m:math>
                    <m:apply
                    xmlns:i="http://www.cs.unibo.it/helm/InputNotationalElements"
                    xmlns:o="http://www.cs.unibo.it/helm/OutputNotationalElements">
                      <m:leq/>
                      <m:ci>O</m:ci>
                      <m:apply>
                      <m:plus/>
                        <m:cn>1</m:cn>
                        <m:ci><m:msub><m:mi>n</m:mi><m:mn>0</m:mn></m:msub></m:ci>
                      </m:apply>
                    </m:apply>
                  </m:math>
                </omdoc:FMP>
                <omdoc:method xref="Apply">
                  <m:math>
                    <m:ci>le_S</m:ci>
                  </m:math>
                  <m:math>
                    <m:ci>.</m:ci>
                  </m:math>
                  <m:math>
                    <m:ci>.</m:ci>
                  </m:math>
                  <omdoc:premise xref="i32">
                    <m:math>
                      <m:ci>IHn</m:ci>
                    </m:math>
                  </omdoc:premise>
                </omdoc:method>
              </omdoc:derive>
            </omdoc:proof>
          </omdoc:method>
        </omdoc:derive>
      </omdoc:proof>
    </omdoc:method>
  </omdoc:derive>
</omdoc:proof>
```

B MathML-Presentation

```
 <?xml version="1.0"?>
<m:math
xmlns:m="http://www.w3.org/1998/Math/MathML"
xmlns:helm="http://www.cs.unibo.it/helm"
xmlns:xlink="http://www.w3.org/1999/xlink">
  <m:mtable>
    <m:mtr>
      <m:mtd>
        <m:mrow>
          <m:mtext>DEFINITION cic:/Coq/Arith/Le/le_O_n.con(</m:mtext>
          <m:mtext>) OF TYPE</m:mtext>
        </m:mrow>
      </m:mtd>
    </m:mtr>
    <m:mtr>
      <m:mtd>
         <m:mrow>
          <m:mphantom>
            <m:mtext>__</m:mtext>
          </m:mphantom>
          <m:mrow>
            <m:mrow>
              <m:mo mathcolor="Blue">&#x2200;</m:mo>
              <m:mi>n</m:mi>
              <m:mo>:</m:mo>
              <m:mi>nat</m:mi>
              <m:mo>.</m:mo>
              <m:mrow>
                <m:mi>O</m:mi>
                <m:mo>0/00?</m:mo>
                <m:mi>n</m:mi>
              </m:mrow>
            </m:mrow>
          </m:mrow>
        </m:mrow>
      </m:mtd>
    </m:mtr>
    <m:mtr>
      <m:mtd>
        <m:mrow>
          <m:mtext>AS</m:mtext>
        </m:mrow>
      </m:mtd>
    </m:mtr>
    <m:mtr>
      <m:mtd>
        <m:mrow>
          <m:mphantom>
            <m:mtext>__</m:mtext>
          </m:mphantom>
          <m:maction actiontype="toggle">
            <m:mtable>
              <m:mtr>
                <m:mtd>
                  <m:maction actiontype="toggle">
                    <m:mtable>
```

```
                        <m:mtr>
                          <m:mtd>
                            <m:mtable>
                              <m:mtr>
                                <m:mtd>
                                  <m:mtext mathcolor="red">assume </m:mtext>
                                     <m:mtext>n</m:mtext>
                                  <m:mtext mathcolor="red">: </m:mtext>
                                  <m:mrow>
                                    <m:mi>nat</m:mi>
                                  </m:mrow>
                                </m:mtd>
                              </m:mtr>
                              <m:mtr>
                                <m:mtd>
                                  <m:mtable>
                                    <m:mtr>
                                      <m:mtd>
                                        <m:mspace width="1em"/>
                                        <m:mtext mathcolor="red">we proceed by induction on </m:mtext>
                                        <m:mrow>
                                          <m:mi>n</m:mi>
                                        </m:mrow>
                                        <m:mtext mathcolor="red"> to prove </m:mtext>
                                        <m:mrow>
                                          <m:mrow>
                                            <m:mi>O</m:mi>
                                            <m:mo>&#x2264;</m:mo>
                                            <m:mi>n</m:mi>
                                          </m:mrow>
                                        </m:mrow>
                                      </m:mtd>
                                    </m:mtr>
                                    <m:mtr>
                                      <m:mtd>
                                        <m:mspace width="2em"/>
                                        <m:mtext mathcolor="red">case </m:mtext>
                                        <m:mtext>O</m:mtext>
                                        <m:mtext> </m:mtext>
                                        <m:mo mathmathcolor="red">&#x21D2;</m:mo>
                                      </m:mtd>
                                    </m:mtr>
                                    <m:mtr>
                                      <m:mtd>
                                        <m:mspace width="3em"/>
                                        <m:mtext mathcolor="red">the thesis becomes </m:mtext>
                                        <m:mrow>
                                          <m:mrow>
                                            <m:mi>O</m:mi>
                                            <m:mo>&#x2264;</m:mo>
                                            <m:mi>O</m:mi>
                                          </m:mrow>
                                        </m:mrow>
                                      </m:mtd>
                                    </m:mtr>
                                    <m:mtr>
                                      <m:mtd>
                                        <m:maction actiontype="toggle">
```

```
                                        <m:mrow>
                                          <m:mspace width="4em"/>
                                          <m:mfenced>
                                            <m:mtext mathcolor="green">Proof of</m:mtext>
                                          </m:mfenced>
                                          <m:mrow>
                                            <m:mrow>
                                              <m:mi>0</m:mi>
                                              <m:mo>&#x2264;</m:mo>
                                              <m:mi>0</m:mi>
                                            </m:mrow>
                                          </m:mrow>
                                        </m:mrow>
                                        <m:mtable>
                                          <m:mtr>
                                            <m:mtd>
                                              <m:mspace width="4em"/>
                                              <m:mtext mathcolor="red">by (</m:mtext>
                                              <m:mrow>
                                                <m:mi>le_n</m:mi>
                                              </m:mrow>
                                              <m:mtext> </m:mtext>
                                              <m:mrow>
                                                <m:mi>.</m:mi>
                                              </m:mrow>
                                              <m:mtext mathcolor="red">)</m:mtext>
                                            </m:mtd>
                                          </m:mtr>
                                          <m:mtr>
                                            <m:mtd>
                                              <m:mrow>
                                                <m:mspace width="4em"/>
                                                <m:mtext mathcolor="red">we proved </m:mtext>
                                              </m:mrow>
                                              <m:mrow>
                                                <m:mrow>
                                                  <m:mi>0</m:mi>
                                                  <m:mo>&#x2264;</m:mo>
                                                  <m:mi>0</m:mi>
                                                </m:mrow>
                                              </m:mrow>
                                              <m:mtext></m:mtext>
                                            </m:mtd>
                                          </m:mtr>
                                          <m:mtr>
                                            <m:mtd>
                                              <m:mspace width="4em"/>
                                              <m:mfenced>
                                                <m:mtext mathcolor="green">hide result</m:mtext>
                                              </m:mfenced>
                                            </m:mtd>
                                          </m:mtr>
                                        </m:mtable>
                                      </m:maction>
                                    </m:mtd>
                                  </m:mtr>
                                  <m:mtr>
                                    <m:mtd>
```

```
173                                     <m:mspace width="2em"/>
174                                     <m:mtext mathcolor="red">case </m:mtext>
175                                     <m:mtext>S</m:mtext>
176                                     <m:mtext> </m:mtext>
177                                     <m:mtext>:</m:mtext>
178                                     <m:mtext> </m:mtext>
179                                     <m:mtext>n</m:mtext>
180                                     <m:mtext>0</m:mtext>
181                                     <m:mtext>:</m:mtext>
182                                     <m:mrow>
183                                       <m:mi>nat</m:mi>
184                                     </m:mrow>
185                                     <m:mtext> </m:mtext>
186                                     <m:mo mathmathcolor="red">&#x21D2;</m:mo>
187                                   </m:mtd>
188                                 </m:mtr>
189                                 <m:mtr>
190                                   <m:mtd>
191                                     <m:mspace width="3em"/>
192                                     <m:mtext mathcolor="red">the thesis becomes </m:mtext>
193                                     <m:mrow>
194                                       <m:mrow>
195                                       <m:mi>O</m:mi>
196                                       <m:mo>&#x2264;</m:mo>
197                                         <m:mrow>
198                                           <m:mn>1</m:mn>
199                                           <m:mo>+</m:mo>
200                                           <m:msub>
201                                             <m:mi>n</m:mi>
202                                             <m:mn>0</m:mn>
203                                           </m:msub>
204                                         </m:mrow>
205                                       </m:mrow>
206                                     </m:mrow>
207                                   </m:mtd>
208                                 </m:mtr>
209                                 <m:mtr>
210                                   <m:mtd>
211                                     <m:mspace width="3em"/>
212                                     <m:mtext>(</m:mtext>
213                                     <m:mtext>IHn</m:mtext>
214                                     <m:mtext>) </m:mtext>
215                                     <m:mtext mathcolor="red">by induction hypothesis we know </m:mtext>
216                                     <m:mrow>
217                                       <m:mrow>
218                                         <m:mi>O</m:mi>
219                                         <m:mo>&#x2264;</m:mo>
220                                         <m:msub>
221                                           <m:mi>n</m:mi>
222                                           <m:mn>0</m:mn>
223                                         </m:msub>
224                                       </m:mrow>
225                                     </m:mrow>
226                                   </m:mtd>
227                                 </m:mtr>
228                                 <m:mtr>
229                                   <m:mtd>
230                                     <m:maction actiontype="toggle">
```

```
<m:mrow>
  <m:mspace width="4em"/>
  <m:mfenced>
    <m:mtext mathcolor="green">Proof of</m:mtext>
  </m:mfenced>
  <m:mrow>
    <m:mrow>
      <m:mi>O</m:mi>
      <m:mo>&#x2264;</m:mo>
      <m:mrow>
        <m:mn>1</m:mn>
        <m:mo>+</m:mo>
        <m:msub>
          <m:mi>n</m:mi>
          <m:mn>0</m:mn>
        </m:msub>
      </m:mrow>
    </m:mrow>
  </m:mrow>
</m:mrow>
<m:mtable>
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